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# SAFETY-BASED GUIDELINES FOR LEFT-TURN PHASING DECISIONS WITH NEGATIVE BINOMIAL REGRESSION

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SAFETY-BASED GUIDELINES FOR LEFT-TURN PHASING DECISIONS WITH  
NEGATIVE BINOMIAL REGRESSION

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THESIS

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A thesis submitted in partial fulfillment of the requirements for the degree of Master  
of Science in Civil Engineering in the College of Engineering at the University of  
Kentucky

By

Kiriakos Amiridis

Lexington, Kentucky

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Lexington, Kentucky

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## ABSTRACT OF THESIS

### SAFETY-BASED GUIDELINES FOR LEFT-TURN PHASING DECISIONS WITH NEGATIVE BINOMIAL REGRESSION

The efficient and safe movement of traffic at signalized intersections is the primary objective of any signal phasing and timing plan. Accommodation of left turns is more critical due to the higher need for balancing operations and safety. The objective of this study is to develop models to estimate the safety impacts of the use of left-turn phasing schemes. The models are based on data from 200 intersections in urban areas in Kentucky. For each intersection, approaches with a left-turn lane were isolated and considered with their opposing through approach in order to examine the left-turn related crashes. This combination of movements is considered to be one of the most dangerous in terms of intersection safety. Hourly traffic volumes and crash data were used in the modeling approach along with the geometry of the intersection. The models allow for the determination of the most effective type of left-turn signalization based on the specific characteristics of an intersection approach. The accompanying nomographs provide an improvement over the existing methods and warrants and allow for a systematic and quick evaluation of the left-turn phase to be selected. The models utilize the most common variables that are already known during the design phase and can be used to determine whether a permitted or protected-only phase will suit the intersection when considering safety performance.

**KEYWORDS:** Left-Turn Phasing Decisions & Nomographs, Negative Binomial Regression, Signalized Intersections, Crash Data Analysis, Road Safety

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Tuesday, January 17 2017

SAFETY-BASED GUIDELINES FOR LEFT-TURN PHASING DECISIONS WITH  
NEGATIVE BINOMIAL REGRESSION

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To my beloved grandfather,

Konstantine Paschalidis

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# 1 INTRODUCTION

A fundamental objective of traffic signals is the development of phasing and timing plans that improve efficiency of operations and reduce delays while maintaining a high level of safety. One issue of concern is the treatment of left-turn phasing, which can operate as protected-only, permitted (yielding to conflicting traffic) or a combination permitted/protected movement. While protected-only phasing can improve safety, it can also increase delays and congestion at the intersection. Permitted movements can safely serve traffic when volumes are low, such as during off-peak periods, but may experience safety or capacity problems with high volumes, such as during the AM and PM rush hour. The recent introduction of the Flashing Yellow Arrow provides the opportunity to vary left turn phasing by time of day further complicating the selection of appropriate phasing. Current guidance has not yet evaluated the effect of hourly variations as most safety performance models focus on Average Daily Traffic Volumes, and operational models focus peak hour demand. Furthermore, Stamatiadis et al. (1) in their recent research have called into question, the validity of threshold conditions promoted by common practices, such as the cross product methodology. Therefore, there is a need for improving existing guidelines for the use of left-turn phasing to provide improved selection by time of day to deliver safe and efficient operations through varying traffic demands.

Signalized intersections are a critical component of the roadway system and frequently act as choke points on the transportation system. As an example, signalized intersection crashes account for approximately 26 percent of all crashes in Kentucky (2). Left-turning maneuvers are considered as one of the most hazardous traffic movements, since turning vehicles have to cross in front of the opposing through traffic. The difficulty of completing this movement is evident in crash statistics indicating that 45 percent of all crashes that occur at intersections throughout the United States involve left-turning vehicles even though left-turning movements represent a disproportionate small percentage (10-15 percent) of all the approach traffic (3). To alleviate this problem and improve safety, protected left-turn phasing is frequently installed at traffic signals.

The issue of left-turn phasing is a two-step process. The first question is whether an exclusive left-turn phase is warranted. Major factors affecting this decision are peak hour left-turn and opposing volumes, left-turn delays, and left-turn crashes. After a decision is reached to add a left-turn phase, one of two basic phasing methods is commonly used:

- 1) protected-only, where the driver is allowed to turn left only during the green arrow portion of the cycle while the opposing traffic is stopped; or
- 2) a combination of protected and permitted left-turn phasing, where during a portion of the left-turn phase the left-turning movement is protected from opposing traffic but drivers can continue to turn left during the remaining green through phase when there are available gaps in the opposing traffic.

In addition to the factors affecting the decision for the installation of left-turn phasing, a constant trade-off between the goals of efficiency and safety is present and thus, influences the final decision.

There are no nation-wide acceptable criteria or prediction models for the installation and usage of left-turn phasing despite the fact that studies exist that have developed guidelines for the use of left-turn phasing. Most the current state policies prescribe the use of protected-only phasing for certain geometric configurations, such as when three or more opposing through lanes are present, when dual left-turn lanes exist, if there is insufficient sight distance for the turning vehicle and opposing traffic, or if the intersection geometrics prevent adequate sight distance due to lane configuration and offsets. Additionally, the common ground of the existing guidelines is the use of traffic volumes and threshold values for crashes and acceptable delays as means to make a decision. Moreover, each state has its own criteria in determining when a severe crash problem occurs and when a left-turn treatment is needed or warranted.

The objective of this study is to develop models that can utilize readily available information to determine the potential safety performance of left-turn phasing schemes. This will allow for a systematic evaluation of the various schemes and provide decision-makers with a tool to evaluate options before determining the option to be used. It is expected that the findings of this research will be used to improve intersection operations and assist in creating a more appropriate left-turn phasing guidance for varying traffic demand.

## 2 LITERATURE REVIEW

This literature review briefly discusses current research findings and reviews policies of other state agencies relative to permitted left-turn guidelines.

### 2.1 Guidelines

In 1979, Agent developed one of the first efforts addressing protected left-turn phasing. He proposed a set of warrants for intersections with a left-turn lane that were based on crash experience, delays, volumes, and traffic conflicts (4). The warrants were based on a set of Kentucky intersections and state practices at the time of the research. These warrants were evaluated and augmented with guidelines for permitted/protected left turns in 1985 (5). Agent found that a considerable increase in left-turn crashes occurred when permitted/protected phasing replaced protected-only phasing when the cross product was above 50,000 for one opposing single lane and 100,000 for two opposing lanes. In 1982, the Florida Section of the Institute of Transportation Engineers (FL-ITE) conducted a before and after crash analysis of intersections that were converted from protected-only to permitted/protected as well as those with a reverse change, i.e., from permitted/protected to protected-only (6). The study utilized this before and after crash analysis along with a survey of FL-ITE members to develop a set of guidelines for left-turn phasing selection. The guidelines developed were very similar to those developed by Agent (5). Cottrell (7) also developed a set of guidelines in an effort to address this issue for the Virginia DOT in 1985. These guidelines were similar to the ones developed by Agent and FL-ITE.

Several states consider a combination of criteria to determine whether a left-turn phase is required. For example, Arizona (8) and California (9) use cross product, left-turn volume, delay of left turns, and crash history while Indiana (10) uses left-turn volume and delays and Virginia (11) uses cross product and crash history. It should be noted though that these are not combined into a single criterion but rather left-turn phasing decisions can be based on any single criterion. Even though several states use similar guidelines, there is no agreement on the threshold values to be used when a left-turn phasing decision is required. For example, the use of cross product threshold value varies among the states using this criterion. In this case, Virginia uses 50,000, California 100,000, Arizona 50,000-225,000 depending on lane configuration and intersection location (urban/rural), Oregon 150,000 or 300,000 depending on the number of opposing lanes and phasing type (12) and Texas 130,000 or 93,000 per lane based on number of opposing lanes (13).

Stamatiadis et al. (14) considered delays and crashes in developing guidelines and boundary conditions for selecting the appropriate left-turn phase. The study utilized micro-simulation for operational decisions and crash history for safety and developed nomographs that allow for the selection of the phase type (permitted, permitted/protected or protected-only) based on cross product and left-turn delays or crashes. It should be noted that this was one of the first studies that developed nomographs to be used combining safety and operational criteria as well as considering the impacts of the number of opposing lanes in establishing guidelines for phase. Ozmen et al. (15) developed a process that utilized a Multi-Criterion Decision Analysis in selecting appropriate left-turn phase. The approach

developed considers volumes, geometry and crashes while ranking possible left-turn phasing options. Their approach provided an index-based recommendation using weights and scores resulting in a numerical scale for comparing each type of left-turn control with the others instead of an absolute type.

## **2.2 Safety**

The safety of left-turning vehicles has been the topic of past research that resulted in developing guidelines for the installation of left-turn phasing (4, 6, 14, 16, 17, 18, 19). These studies use two distinct methods, empirical analysis and microsimulation.

Past research has indicated that the intersection features that affect safety and are prominent in determining the left-turn treatment include traffic volumes (opposing through, left-turning, and their cross product), geometry (number of opposing lanes and presence of exclusive left-turn lanes), and operational characteristics (speed limits, sight distance, and delays). Among these features, traffic volumes are more widely used by establishing upper limits for specific phasing treatment. The number of left-turn related crashes has also been used in determining the left-turn phasing (14, 20).

There have been a number of efforts to develop Crash Modification Factors (CMFs) in order to estimate the safety effect of left-turn phase options and changes. Hauer (21) reviewed 14 studies conducted over a 24-year period and concluded that the CMF for converting from permitted to protected left-turn phase most likely depends on the number of opposing lanes and that most of the other evidence is insufficient and contradictory. Hauer estimated that the CMF for changing to protected-only phasing from either permitted only or permitted/protected is approximately 0.30 for left-turn crashes. However, he noted that for total crashes the CMF is 1.0, i.e., no effect. Hauer argued that a change to protected-only phase from a permitted/protected left-turn phasing will substantially reduce left-turn crashes but would have no difference in the total number of crashes, due to increased delay and congestion at the intersection.

Harwood et al. (22) conducted a before-after study using the empirical Bayes (EB) approach to study the safety impact of adding left-turn lanes with protected-only or permitted/protected signal phasing. A total of 36 four-leg signalized intersections were included; 31 of these sites received a permitted/protected signal phasing while 5 received a protected-only signal phasing. The 31 sites with permitted/protected signal phasing system experienced a 9 percent reduction in crashes (CMF of 0.91); the five sites with protected-only signal phasing system experienced a 10 percent reduction in crashes (CMF of 0.90). The study report did not indicate if these results were statistically significant. The authors conclude that there is “essentially no effect of the type of signal phasing on the safety effectiveness of left-turn lanes”, and “there are too few data to obtain definitive results”.

Srinivasan et al. (23) conducted a study to determine the safety effect of converting left-turn phasing schemes from one type to another. Their study considered changes to protected-only phasing from either permitted only or permitted/protected. Their findings

were very similar to those noted by Hauer (21). The study indicated that the lack of overall crash reduction from such phase changes could be attributed to potential increase of rear end crashes. However, the authors indicate that the overall effect could be positive if one considers potential differences in severity between left-turn and rear end crashes. Even though their study examined conversions from permitted to permitted/protected phasing, there were no recommendations because the sample was very small.

In a more recent effort, Srinivasan et al. (24) attempted to develop a CMF for left-turn phasing conversions based on a large number of intersections in North Carolina and Toronto. The study considered intersections that converted from permitted to permitted/protected and used an Empirical Bayesian approach to estimate the CMFs from such change. Safety Performance Functions (SPFs) were estimated for crash severity (injury), total number of crashes and crash type (left turn, rear end and left turn with opposing through). The study showed that target crashes, i.e., left-turn related, improve with the change and when more than one approaches is treated with the change, there is an overall crash reduction. However, the total number of crashes increases with the change and this could be attributed to the increase in rear end crashes. One issue with this research is that exposure metrics used to develop these CMF are all based on Average Daily Traffic Volumes and do not account for the effect of left turn volume on crash exposure as was documented by Agent (5), FL-ITE (6) and Cottrell (7).

The studies reviewed here show a general trend in decreased left-turn crashes with protected-only left-turn phasing. However, they do not provide the guidance necessary to identify crash performance as a function of other operational parameters, nor do they provide the resolution to select left turn treatments based on hourly variations in turn volume and directionality to assist in the development of time of day signal phasing. In order to develop such guidance, crash analysis must be approached differently identifying crash modification functions to determine the rate of reduction as a function of operational parameters or through direct analysis of the safety performance function to identify when predicted crashes increase to an unacceptable level.

### **3 METHODOLOGY**

In order to address the need for detailed left-turn volume in left-turn phasing selection, this research set to develop hourly left turn crash prediction models based on data typically available during signal retiming projects. This included intersection geometry and hourly turning movement counts at the intersection. Crash data was then disaggregated by hour to develop unique data points of geometry, volume, and crashes for each observed hour. Based on this dataset explanatory statistical models were developed to allow understanding of the influence of recorded factors so that necessary guidance could be developed.

#### **3.1 Database Development**

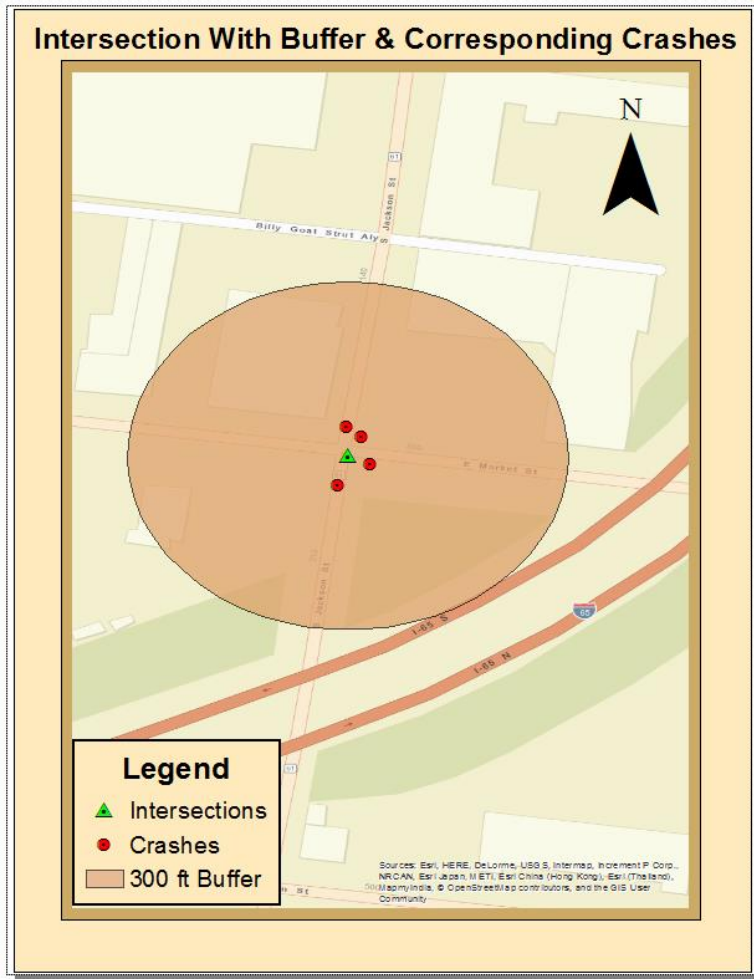
Hourly traffic volumes were obtained for a total of 200 actuated signalized intersections mainly in the areas of Lexington and Louisville, Kentucky. Counts ranged from 2-hour AM and PM peak hour counts to 24-hour turning movement counts collected by each agency for this study. The type of left-turn phasing scheme, i.e., permitted, protected or permitted/protected, was identified based on the type of signal installation at the intersection. For each intersection, the number of lanes and their use (i.e., left, through and right or combinations) of each lane by approach was determined. The information regarding the type of phasing scheme and intersection geometry was derived through observation from Google Earth. These two data types allow for the examination of the potential contribution of geometry and phasing scheme on the intersection crashes.

The crash history of each intersection was obtained for the 6-year period, 2010-2015, through the “Kentucky Collision Analysis for the Public” based on specific filters (25). Each crash was evaluated to determine whether it was left-turn related with opposing traffic based on the crash type and specific directions of the vehicles involved. This was achieved by selecting the pre-collision vehicle action code as either going straight ahead or turning left and the crash type as angle collision (one vehicle turning left), rear end (one vehicle turning left), or opposing left turn. This process identified only the pertinent crashes that could be related to left-turn phasing and eliminate all others that could create noise in the dataset. For each crash, the directions of the vehicles were recorded in order to determine the left turn and opposing through combination of the approaches to be used in the analysis.

The next step in the crash database development was the examination of the time the crash occurred in order to “join” them with the available hourly volumes and to ensure that the crash occurred within the specific time period provided. This process resulted in utilizing 756 crashes in 7,677 approach combinations. Among these 7,677 approach combinations, there were 3,111 with a permitted, 2,441 with a permitted/protected, and 2,125 with a protected-only phase.

In order to relate the crashes with their corresponding intersections, the “Spatial Join” command was applied by using the ArcGIS software. A 300ft buffer was created in each intersection, indicating that each crash contained in that buffer is related to that specific intersection. The results of this procedure for a sample intersection in the city of Louisville are presented in Figure 1.





**Figure 1: Sample Intersection Showing the 300 ft Buffer and Corresponding Crashes**

### 3.2 Variable Selection

The first step in the analysis focused on identifying variables that could be used in the models. Table 1 shows the range of values for each of the variables available in the database. Some of the left-turn volumes are very small and this is due to traffic counts conducted in early morning hours (e.g. 2:00-5:00 am).

**Table 1: Data Range Values**

Variable	Minimum	Maximum	Mean
Left-turn volume (vph)	1	850	75
Through volume (vph)	1	2364	338
Number of opposing through lanes	1	4	1.518
Number of crashes	0	6	0.098

In order to determine the number of opposing through lanes, all lanes that serve movements, i.e., through and right turns, in conflict with the left turn were included. Most

of the approach combinations had a single opposing through-related lane (53.2 percent) or two lanes (42.0 percent). There was a small number of approach combinations with three lanes (4.6 percent) and a few with four lanes (0.2 percent).

Most of the approach combinations (91.6 percent) had no crashes within the respective hourly time period, only 7.2 percent of the combinations had one crash, whereas there were a few approaches with more than one crash accounting for 1.2 percent of the total.

## 4 STATISTICAL FRAMEWORK

As noted above, the initial database will be separated in three sub-databases each representing one type of left-turn phase: Permitted-only, Permitted/Protected, and Protected. Therefore, in this observational study there is one categorical explanatory variable/factor (type of left-turn phase) with three levels. Besides the categorical explanatory variable, there are three quantitative explanatory variables that will be analyzed in the model:

1. Hourly Volume of Left Turns Per Approach
2. Respective Hourly Opposing Through Volume Per Approach
3. Number of Opposing Through Lanes.

These three quantitative explanatory variables will not be inserted the model separately, but as a new variable that combines them into one, i.e., considers their interaction. In other words, there will actually be one quantitative explanatory variable in each of the three models which will correspond to the product of the three quantitative explanatory variables mentioned above. The justification of this decision that was made follows.

In terms of crash occurrence, the literature indicates that there is an interaction between volumes turning left and respective opposing through volumes. The effect that the left-turning volume has on a crash occurrence is dependent upon the respective opposing through volumes. For example, the magnitude of the effect that a left-turning volume of 200 vehicles per hour has on the crash occurrence is different whether the respective opposing volume is 300 or 1,000 vehicles per hour, meaning that 1,000 vehicles per hours are expected to result in more crashes in the long run. After all, in order for a crash to occur there must be, by definition, a combination of left and opposing through movement. Therefore, the main concern in this study is to examine whether the interaction between the left-turning volumes and the respective opposing through volumes is statistically significant. Ideally, it would be preferred that besides the interaction term, the main effects, i.e.,  $V_L$  and  $V_{Th}$ , would also enter the model as separate explanatory variables. However, as it will be discussed in the sections that follow, the main effects are not statistically significant in neither of the three models.

The reason that it is preferred to include the main effects in a prediction model, in general, is that one can differentiate the impact of each main effect on the predicted variable. Therefore, in these models where only the interaction term is included, no interpretation of main effects is possible and it cannot be argued whether the effect of one variable is larger than the other. The unique information of each explanatory variable is “lost” by including the interaction term without its main effects. At this point it must be emphasized that models where interaction terms are included without their respective main effects is statistically acceptable as long as the purpose of the model is prediction. Indeed, the purpose of the models to be developed here and presented in the form of nomographs is the prediction of the number of crashes and only, i.e., prediction models.

The number of opposing through lanes will be added in the interaction term of the model. This allows for testing whether the magnitude of the effect of the interaction between  $V_L$  and  $V_{Th}$  is different depending on the number of opposing through lanes when crash occurrence is concerned. Therefore, a 3-way interaction will be tested for each of the three models:

$$V_L \times V_{Th} \times N$$

where:

$V_L$  : Volume of Left-Turns

$V_{Th}$  : Respective Opposing Through Volume

$N$  : Number of Opposing Through Lanes

Before continuing further in the analysis, the number of opposing through lanes must be examined in more detail. The potential percentage of the number of opposing lanes for each type of left-turn scheme is presented in Table 2.

**Table 2: Percentage of Number of Opposing Lanes Per Left-Turn Scheme**

Number of Opposing Through Lanes	Type of Left-Turn Phase			Total	Percent
	Permitted-only	Permitted/Protected	Protected		
1	2504 (80.5%)	937 (38.4%)	643 (30.3%)	4084	53.2%
2	484 (15.6%)	1492 (61.1%)	1249 (58.8%)	3225	42.0%
3	123 (4.0%)	12 (0.5%)	221 (10.4%)	356	4.6%
4	0 (0%)	0 (0%)	12 (0.6%)	12	0.2%
Total	3111	2441	2125	7677	100%
Percent					

Table 2 indicates that 3 and 4 opposing through lanes are under-represented in the dataset and therefore it would not be acceptable to include them in the model. Therefore, it was decided to conduct the analysis by excluding the cases where the number of opposing through lanes was 3 or 4.

It is possible to develop a prediction model simply utilizing the entire dataset without splitting it into three sets based on the left-turn phase. In this case, the type of the left-turn scheme is included in the model through the insertion of two dummy variables, since the factor has three levels. This is viable and frequently used in statistical analyses and ideally, this type of analysis would be preferred. However, in this case, the unequal sample sizes across the three left-turn schemes (40.5% for Permitted-only, 31.8% for Permitted/Protected, and 27.7% for Protected only) results in issues for the analysis in terms of non-independence, non-orthogonality and confounding variables. Therefore, it was decided to run separate analyses for each phase scheme and at the end provide an interpretation in terms of practical significance on the three models. More specifically, it

will be examined whether it was indeed pertinent to separate the dataset based on the left-turn scheme through an examination as to whether the differences indicated by the three models are actually significant in the context of the literature and a priori expectations in general.

#### 4.1 Selection of the Appropriate Generalized Linear Model

The response variable “Number of Crashes” is a count random variable that measures/counts the number of crashes that occurred in one hour regarding the left-turn/opposing movement combination. Therefore, in a 4-leg intersection where all turns are permitted, there would be four such movement combinations.

The dependent variable (number of crashes) corresponds to count data and therefore the most common distributions that are utilized are the Poisson and the Negative Binomial distributions which are both generalized linear models (GLM) and the log link function will be applied in both cases. At this point it should be mentioned that the Zero Inflated Poisson (ZIP) or the Zero Inflated Negative Binomial (ZINB) can be also used here. The Zero Inflated models should be utilized when there is no possibility of having a crash in certain circumstances based on the model formalization. For example, if there were zero left or through volumes, then it could be reasonably argued that there is no chance of having a crash simply because there are no vehicles present during that specific time period in that specific intersection approach. However, these scenarios have been adjusted for in the model, since only cases where crashes can potentially occur have been included. Therefore, there is not an excess of zeros (also defined as “structure zeros”) that would suggest any type of a zero inflated model. Therefore, Zero Inflated models are not appropriate here and the Poisson and Negative Binomial models are evaluated next.

The two essential parameters in the Poisson and Negative Binomial distributions are the mean and the variance. Their underlying assumptions for application in a database are based on these two parameters. At this point a brief overview of the Poisson and Negative Binomial regressions would be useful to support the decisions that will follow.

All generalized models are based on two crucial functions: “the link function that relates the mean  $\mu = E(y)$  to the linear predictor  $X\beta$  and the variance function that relates the variance as a function of the mean  $V(y) = \alpha(\varphi)v(\mu)$  where  $\alpha(\varphi)$  is the scale factor. For the Poisson, Binomial, and Negative Binomial variance models,  $\alpha(\varphi) = 1$ ” [reference].

It is noted that  $X$  regards the vector of the factors/covariates, whereas  $\beta$  corresponds to the vector of coefficients.

The generalized linear models (GLMs) underlying assumptions are the following [reference]:

1. Statistical independence of the  $n$  observations.
2. The variance function  $V(y)$  is correctly specified.
3.  $\alpha(\varphi)$  is correctly specified (1 for Poisson, binomial, and negative binomial).

4. The link function is correctly specified.
5. Explanatory variables are of the correct form.
6. There is no undue influence of the individual observations on the fit.

For both the Poisson and Negative Binomial Count Models, and especially when the response variable refers to crash data, the most appropriate link function is the (natural) log link. In fact, for the Negative Binomial model, the most common parameterization of the link function is the log link. The natural logarithm of the response variable is expressed as a linear combination of the explanatory variables with their respective coefficients:

$$LN(\text{Number of Crashes}) = \mathbb{X}\beta$$

It is noted that this form of the GLM will be applied if either the Poisson or Negative Binomial Model will be proven to be more appropriate. The next step involves the determination of which of these two models is more appropriate for the given database. Table 3 presents the variance function for each distribution and it indicates that the Poisson Variance Function can be realized as a special case of the Negative Binomial Variance Function. More specifically, if  $k = 0$ , then the Negative Binomial variance is exactly equal to the Poisson.

**Table 3: Variance Functions for Poisson and Negative Binomial Distribution**

Family (Distribution)	Variance Function $V(\mu)$	Range Restrictions	$\frac{\partial V(\mu)}{\partial \mu}$
Poisson	$\mu$	$\begin{cases} \mu > 0 \\ y \geq 0 \end{cases}$	1
Negative Binomial	$\mu + k\mu^2$	$\begin{cases} \mu > 0 \\ y \geq 0 \end{cases}$	$1 + 2k\mu$

Theoretically, in a perfect Poisson distribution, the variance is equal to the mean. However, in a real dataset, this is highly unlikely to be the case; the important question is how much they differ. Depending on the measure of this difference as well as its sign (positive or negative), other distributions or techniques may be more appropriate to consider. If the variance is larger than the mean, then the dataset is characterized as overdispersed, whereas if the variance is less than the mean then the dataset is characterized as underdispersed. It is noted that overdispersion often appears when there is a large number of zeros in the dataset; this is the norm when the dataset corresponds to crash data counts as in this case. However, the Negative Binomial distribution, can take into account the overdispersion in a dataset since it has an additional parameter (dispersion parameter) that is used to model the variance. In other words, an alternative strategy of modeling overdispersed data that follow a Poisson distribution is the negative binomial distribution. The mean and variance for each of the three types of phasing schemes are presented in Table 4.

**Table 4: Mean and Variance Comparison for Each Type of Left-Turn Phasing Scheme**

Type of Left-Turn Phase Scheme	Mean (1)	Variance (2)	Difference (2)-(1)	Difference %
Permitted	0.077	0.090	0.013	16.9%
Permitted/Protected	0.134	0.176	0.042	31.3%
Protected-only	0.088	0.122	0.034	38.6%

The data in Table 4 shows that for all three left-turn phasing schemes the dataset is overdispersed. Therefore, the Negative Binomial distribution fits the data and the recommended models will be assumed to follow a negative binomial distribution. The Negative Binomial distribution is also the most common probability distribution used in transportation safety analyses for modeling motor vehicle crashes (21, 23, 29, 30).

Therefore, overdispersion is a typical phenomenon when dealing with crash data due to the excess of zeros. In the cases where overdispersion is present, the most common “countermeasure” is to apply the Negative Binomial model instead of the Poisson. The reason that the Negative Binomial distribution is advised to be utilized is because that extra term  $k\mu^2$  in its variance can accommodate the non-equality between the mean and the variance. The Negative Binomial can be realized as a distribution that has one additional degree of freedom when it comes to fitting data, compared to Poisson which is a one-parameter distribution.

Clearly, in real case scenarios it is impossible to obtain an exact equality between the mean and the variability. Therefore, one might argue that when dealing with count data where overdispersion is present the Negative Binomial regression can be utilized without even considering the Poisson since the Negative Binomial is a generalization of the Poisson. However, this is not a suggested practice and should be avoided. Whether the Negative Binomial regression should be utilized or the Poisson can be tested through a statistical test. The statistical test is conducted on the variance function of the Negative Binomial distribution. The statements of the null and alternative hypotheses follow:

Null Hypothesis:  $k = 0$ ; The Mean is equal to the Variance (Therefore a Poisson distribution should be used)

Alternative Hypothesis:  $k > 0$ ; The mean  $\mu$  is greater than the Variance (Therefore a Poisson regression should not be utilized)

Failure to reject the null hypothesis actually implies that there is not statistically significant evidence at the significance level of  $\alpha = 0.05$  to conclude that the Poisson regression should have been used instead of the Negative Binomial. It is noted that the statistical tests for each model will be conducted in SAS.

To recapitulate, all of the models will be initially run by utilizing the Negative Binomial regression and at the end it will be examined through a statistical hypothesis test whether this was indeed a better choice compared to the Poisson regression. If the null hypothesis is

rejected, then the whole analysis would have to be conducted once more by assuming a Poisson regression.

Another valuable concept that can be utilized in GLMs is the idea of an offset. The offset is a non-stochastic/deterministic term that can be placed in the model. If an offset is placed in a GLM, then the final regression equation would be:

$$LN(\text{Number of Crashes}) = \mathbb{X}\beta + \text{offset}$$

The offset term is especially important when a conversion over the unit of time or space is necessary. The logic is exactly the same as described in the Poisson Process [reference ghahramani] where the mean/rate  $\lambda$  can be converted from one time/space scale to another interchangeably. The crash data obtained for this analysis is based on a 6-year period and therefore the final mean of the number of crashes should be divided by 6. Therefore, the offset should be set to be  $LN(6)$ ; by implementing this offset, the final form of the GLM would become:

$$LN(\text{Number of Crashes}) = \mathbb{X}\beta + LN(6)$$

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = \mathbb{X}\beta$$

## 4.2 Framework of the Statistical Analysis

The first modeling efforts used the left-turn volume ( $V_L$ ) and the corresponding opposing volume ( $V_{Th}$ ) as predictor variables based on the literature review findings. The model used the cross product of these variables and the phasing type was also included as a factor. It should be noted here that separate models are determined for each left-turn phasing scheme as it was determined before. Stamatiadis et al (28, 29) in prior research had indicated that there is a difference in significance for conflict contribution between the left turn and its associated opposing through volume. The same research also indicated that the effect of the number of lanes is multiplicative and therefore a new analysis was undertaken to determine the possibility of a model where the left and opposing through volumes were used in conjunction with the number of lanes.

In this section, the framework of the statistical analysis to be followed in each model is described in a step-by-step process:

1. Analyze each model by simply including the 3-way interaction term and examine whether the model is statistically significant.
2. Include the main effects,  $V_L$  and  $V_{Th}$  and examine whether they are statistically significant as well if the first step provides statistically significant results.



3. Indicate potential influential data points and decide whether they should be excluded from the data set.
4. Re-run the model if influential points are excluded from the data set, obtain the new estimations of the coefficients and verify that the model remains statistically significant with the exclusion of the influential points.
5. Check the underlying assumptions of the negative binomial regression.
6. Check whether the Negative Binomial regression was indeed more appropriate than the Poisson regression.

This is the basic framework of the statistical analysis and the study in general. However, some more detailed information of the procedures regarding influential data points identification and assumptions assessment of the models are presented. These concepts will be applied in each of the three models.

#### **4.2.1 Unusual and Influential Data**

An (individual) observation is influential if by excluding it from the regression model, the parameter estimates of the coefficients considerably change. In a regression model, an outlier is an individual observation that has a relatively (compared to the other observations) large residual. In other words, an outlier is an individual observation with unusual  $y$  i.e., prediction, value. Leverage points in a regression model are considered observations that have an extreme value on a predictor variable. In other words, a leverage point is an individual observation with an unusual  $x$  value on a predictor variable. Finally, an influential point is an individual observation that is both an outlier and a leverage point, meaning that if that particular point is excluded from the data then the parameter estimates of the coefficients will substantially alter. The change in the parameter estimates if an observation is removed from the data can be calculated with Cook's distance or simply Cook's D.

“There are two different schools of thought about how Cook's D statistic should be used:

1. The values should be compared to some absolute cutoff
2. Do not use absolute cutoffs. Simply pick out those observations whose Cook's D values (if any) differ appreciably from most of the values.

The second approach is used here. Much recent research has shown that comparison to absolute cutoffs is not as effective in identifying influential observations as examining observations with unusually large Cook's D values” (31).

It should be also noted that if influential points are identified, they are not automatically excluded from the dataset. Actually, it is these influential points that sometimes might provide valuable information about the dataset and reveal concepts that may have not been

considered during the initial design of the study. Therefore, if an influential point is identified, then this means that additional examination of this particular point must be conducted in order to determine whether it should be excluded. In addition, the final decision must also take into consideration the content of the phenomenon that is being studied.

Since the influential points will be determined according to Cook's distance, respective plots will be provided for each model and discussed.

#### **4.2.2 Assessing the Assumptions of the Regression Model**

The assessment of the assumptions in any regression model is actually the concept that gives value to the statistical analysis allowing for inference about the population. In an ordinary linear regression, a common practice is to check the underlying assumptions through residual plots; there are also statistical tests that can test the assumptions as well, but a residual plot can give more valuable information in terms of identifying patterns and understanding potential problems with the data. However, in the case of GLMs, the appropriate strategy of assumption assessment is still an open issue since there are many types of residuals and no explicit answer has been given in terms of which has to be used for each case and actually what has to be observed in the residual plots. On the other hand, it is also true that some basic guidelines are generally accepted which are mainly related to the goodness of fit (GOF).

Based on the literature review, the most compact explanation in terms of what is assumed, and why, in a GLM is provided below:

“Ordinary least squares (OLS) extends Maximum Likelihood (ML) linear regression such that the properties of OLS estimates depends only on the assumptions of constant variance and independence. ML linear regression carries the more restrictive distributional assumption of normality. Similarly, although we may derive likelihoods from specific distributions in the exponential family, the second-order properties of our estimates are shown to depend only on the assumed mean-variance relationship and on the independence of the observations rather on a more restrictive assumption that observations follow a particular distribution” [reference GLM and extensions].

What the latter definition actually indicates is that a GLM can be run without assuming anything and after the regression has been applied, it should be tested for independence and whether the assumed mean-variance relationship is indeed the correct one. The correctness of the assumed mean-variance relationship is actually tested based on the GOF.

As previously shown, the null hypothesis of the distribution not following a Negative Binomial could not be rejected, meaning that it is in favor of the GOF. The GOF also implies that the overdispersion of the data has successfully been dealt with in the model through the dispersion factor “ $k$ ” of the Negative Binomial's variance equation. Besides the statistical test, the correctness of the assumed mean-variance relationship can also be verified by the Pearson-based dispersion which is essentially the Pearson Chi-Square

divided by its degrees of freedom. The latter value (i.e., Pearson Chi-Square divided by its degrees of freedom) is equal to one in datasets with zero dispersion. This tests whether the GOF of the Negative Binomial Regression is satisfied; whether the Poisson regression produces even better results in terms of GOF will also be tested through a statistical test.

As far as the assumption of independence is concerned, before attempting to conduct any sort of statistical test to check the assumption, the nature of the data as well as the meaning of independent events must be initially evaluated. More specifically, one must understand what the data stand for and how they were collected. In this case, the data correspond to the number of crashes that occurred in 200 different intersections over a 6-year period. The collected data by itself supports the argument of the independence assumption, since it is highly unlikely that a crash that occurred at a specific point of time and at a specific intersection may affect the occurrence of a crash in another intersection and at a different point of time. Indicatively, 98.8% of the counts in the dataset were zeros and ones meaning that in only 1.2% of the entire 6-year period there were more than one crash that occurred at a specific intersection approach. It is worth mentioning that not even one of these “duplicates” occurred at the same time interval or even day.

## 5 STATISTICAL ANALYSIS

As noted in the previous section, three models will be developed, one for each type of left-turn signal scheme.

### 5.1 Model 1: Permitted-only Left Turn Phasing

#### Step 1: Statistical analysis by including only the 3-way interaction term

The results of the statistical analysis in terms of statistical significance are presented in Table 5.

**Table 5: Parameter Estimates for Initial Model of Permitted-only Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.4508549179	<0.001
$V\_L \times V\_Th \times N$	0.0000082714	0.001

Table 5 indicates that both coefficients are statistically significant at the significance level of 0.05. The form of the model (with a 10-digit decimal precision) would be:

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = -4.4508549179 + 0.0000082714(V\_L \times V\_Th \times N)$$

or alternatively expressed in terms of Number of Crashes:

$$\text{Number of Crashes} = e^{-4.4508549179 + 0.0000082714(V\_L \times V\_Th \times N) + \ln(6)}$$

#### Step 2: Statistical analysis by including the main effects in the model

The results of the statistical analysis in terms of statistical significance are presented in Table 6.

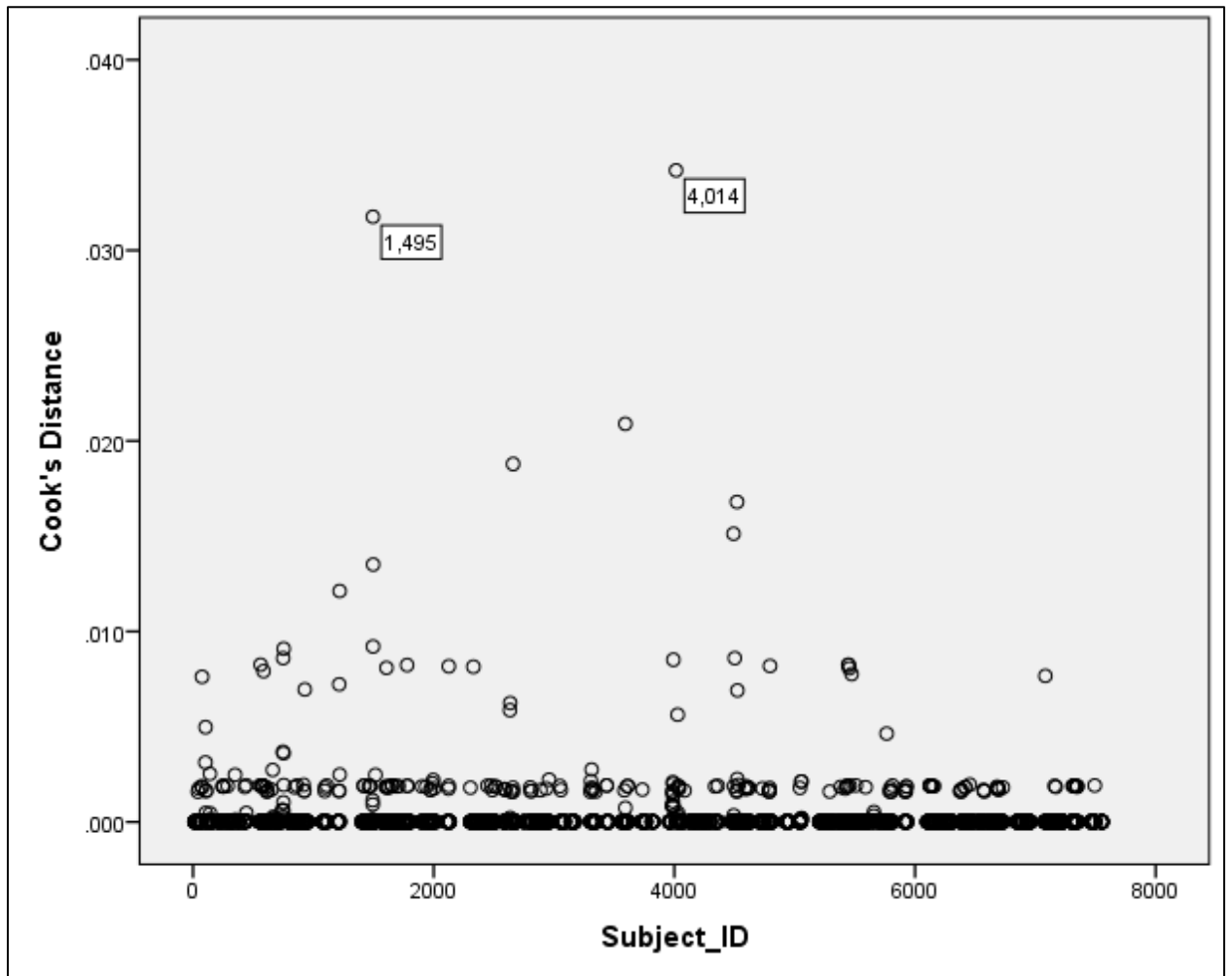
**Table 6: Parameter Estimates for Initial Model of Permitted-only Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.5322878682	<0.001
$V\_L \times V\_Th \times N$	0.0000034611	0.383
$V\_L$	-0.0001442740	0.912
$V\_Th$	0.0008292560	0.046

Table 6 indicates that not all coefficients are statistically significant at the significance level of 0.05. In addition, the sign of the  $V_L \times V_{Th} \times N$  coefficient is negative meaning that the number of crashes decreases while the number of left turn volume and/or opposing through volume increases; this is a result that is counterintuitive and not acceptable based on literature findings and a priori expectations.

### Step 3: Influential Points Identification

The plot of Cook's distance is presented in Figure 2. The influential individual observations, meaning that they differ appreciably from most of the values, which will be further examined in order to decide whether they should be excluded from the dataset are marked on Figure 2.



**Figure 2: Cook's D for Permitted-only Model**

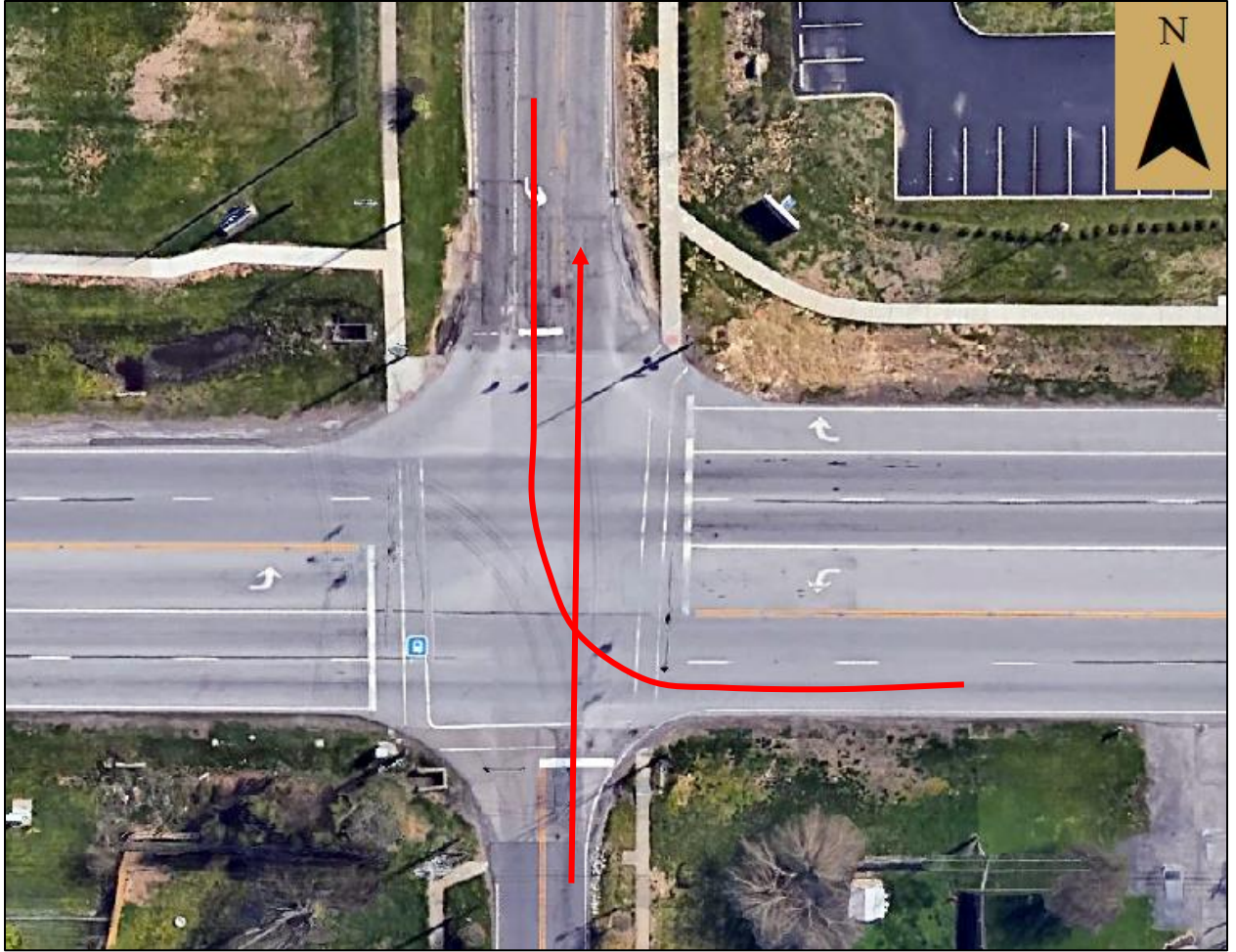
According to Figure 2, there are 2 data points (i.e., 1,495 and 4,014) that are marked whose Cook's D is relatively higher compared to the other observations, meaning that further analysis is required in order to decide whether they should be excluded from the database.

*Analysis of influential point with Subject ID# 1,495*

The descriptive information of the case with Subject ID# 1,495 is presented in Table 7, whereas the aerial image of the intersection is shown in Figure 3.

**Table 7: Crash Related Descriptive Information for Influential Point #1,495**

Subject ID#	1,495
Intersection ID#	153
Direction	Left Turn from North w/ Opposing from South
# of Crashes	1
Hour	18:39
Date	2012-03-11
V_L	224
V_Th	779
Type_LT_P	Permitted-only
N_Opp_Th_Lanes	1
Latitude	38.1938
Longitude	-85.6774



**Figure 3: Influential Data Point with Subject ID# 1,495**

According to Table 7, there does not seem to be anything extreme regarding the explanatory variable. Although the left-turn volume could be considered high it is not considered a reason for exclusion. Moreover, only one crash occurred which cannot be considered an extreme either.

As far as independence is concerned, since there is only one crash occurrence there is not an independence issue.

In addition, Figure 3 does not reveal something unusual regarding the number of lanes or the intersection geometry in general; it seems to be a typical 4-leg intersection.

**Final Decision:** Do not exclude the data point from the analysis.

*Analysis of influential point with Subject ID# 4,014*

The descriptive information of the case with Subject ID# 4,014 is presented in Table 8, whereas the aerial image of the intersection is shown in Figure 4.

**Table 8: Crash Related Descriptive Information for Influential Point #4,014**

Subject ID#	4,014
Intersection ID#	13
Direction	Left Turn from South w/ Opposing from North
# of_Crashes	4
Hour	17:32
	17:33
	17:40
	17:53
Date	2013-11-20
	2013-12-09
	2013-12-29
	2014-12-04
V_L	2
V_Th	86
Type_LT_P	Permitted-only
N_Opp_Th_Lanes	1
Latitude	37.9819
Longitude	-84.4229





**Figure 4: Influential Data Point with Subject ID# 4,014**

According to Table 8, there seem to be some extremes: The number of crashes (4) is relatively high, while on the other hand the left-turning volume is extremely low (only 2 per hour). This might even indicate that there has been an error in data. However, closer inspection reveals that the 3 crashes occurred within one-month period; this cannot have simply occurred by chance since it is extremely rare. Therefore, something was happening during that time period which may have contributed to the large number of crashes; perhaps the road was under construction that period of time volume or a specific event took place. Moreover, this indicates that the events cannot be assumed to be independent. Also, it is worth mentioning that the crashes occurred from 5 to 6 pm which means that the time interval belongs in the PM rush hour period of the day.

Figure 4 indicates that the fact that the number of left-turning vehicles is low is logical since the specific approach (from South) is probably from a farm or from a rural development of some sort. However, the fact that 3 crashes occurred within 1 month

whereas in the remaining 6 –year period only one occurred in addition indicates that the particular time period is biased.

**Final Decision:** Exclude the data point from the dataset due to contradicting values between the response and the explanatory variables and apparent independence among the crashes.

#### Step 4: Re-run the model with excluded influential points

The results of the statistical analysis in terms of statistical significance are presented in Table 9.

**Table 9: Parameter Estimates for Initial Model of Permitted-only Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.4769746934	<0.001
$V\_L \times V\_Th \times N$	0.0000079622	0.001

Table 9 indicates that both coefficients are statistically significant at the significance level of 0.05. The form of the model (with a 10-digit decimal precision) would be:

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = -4.4769746934 + 0.0000079622(V\_L \times V\_Th \times N)$$

or alternatively expressed in terms of Number of Crashes:

$$\text{Number of Crashes} = e^{-4.4769746934 + 0.0000079622(V\_L \times V\_Th \times N) + \ln(6)}$$

#### Step 5: Assumption Assessment

The Goodness of Fit (GOF) statistics are provided in Table 10.

**Table 10: GOF Statistics for Permitted-only Left-Turn Signal Scheme**

	Value	df	Value/df
<b>Deviance</b>	981.636	2984	0.329
<b>Pearson Chi-Square</b>	2975.557	2984	0.997
<b>Log Likelihood</b>	-807.237		
<b>Akaike's Information Criterion (AIC)</b>	1620.473		
<b>Bayesian Information Criterion (BIC)</b>	1638.478		

Table 10 indicates that the Pearson's Chi-square is 2975.557. The critical value for a Chi-square with 2984 degrees of freedom is 3112 at the significance level of  $\alpha = 0.05$ . Therefore, the Pearson's Chi-Square, as it was calculated from the regression model, does not belong in the rejection region since  $2976 < 3112 = \text{critical value}$ ; therefore, there is not statistically significant evidence to reject the null hypothesis. This means that there we cannot conclude that the probability distribution is not a Negative Binomial. Just for the sake of completeness, the corresponding p-value is also calculated as  $p - \text{value} = 0.54 \gg 0.05$  which is much greater than 0.05. Moreover, the dispersion is 0.997 which is almost 1, meaning that practically all of the overdispersion has been accounted for in the model.

#### Step 6: Compare Negative Binomial with Poisson Regression

The statistical test that tests whether the Negative Binomial regression provides a significant improvement in the model compared to the Poisson regression is conducted in SAS and the result is presented in Table 11.

**Table 11: Statistical Comparison Between Negative Binomial and Poisson Regression**

Parameter	Chi-Square	p-value
Dispersion	9.6441	0.0009

According to Table 11,  $p - \text{value} = 0.0009 < 0.05$ ; therefore, there is statistical significant evidence at the significance level of  $\alpha = 0.05$  to reject the null hypothesis and accept the alternative. In other words, the Negative Binomial distribution is a more suitable distribution compared to the Poisson.

#### Nomograph Creation

There will be one nomograph created for the Permitted-only left-turn signal scheme with two distinct curves: one curve corresponds to the case where there is 1 opposing through lane, whereas the second curves corresponds to the case where there are 2 opposing through lanes. In addition, the nomograph assumes one crash per year. Therefore, the nomograph actually corresponds to the following equation:

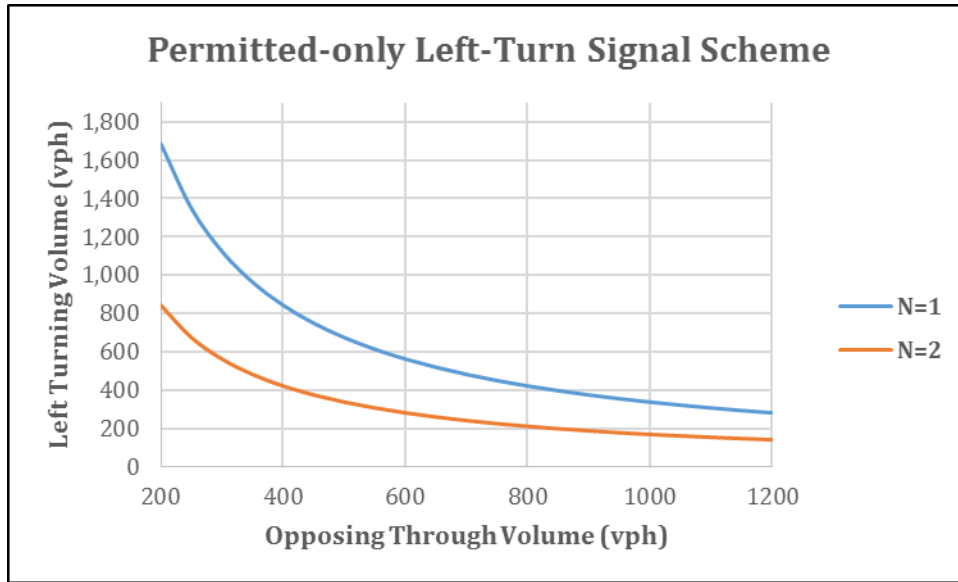
$$\text{Number of Crashes} = e^{-4.4769746934 + 0.0000079622 (V_L \times V_{Th} \times N) + \ln(6)}$$

$$1 = e^{-4.4769746934 + 0.0000079622 (V_L \times V_{Th} \times N) + \ln(6)}$$

$$-4.4769746934 + 0.0000079622 (V_L \times V_{Th} \times N) + \ln(6) = 0$$

$$V_L = \frac{4.4769746934 - \ln(6)}{0.0000079622 (V_{Th} \times N)}$$

The nomograph for the Permitted-only left-turn signal scheme is presented in Figure 5.



**Figure 5: Nomograph for Permitted-only Left-Turn Signal Scheme**

## 5.2 Model 2: Permitted/Protected Left Turn Signal Scheme

### Step 1: Statistical analysis by including only the 3-way interaction term

The results of the statistical analysis in terms of statistical significance are presented in Table 12.

**Table 12: Parameter Estimates for Initial Model of Permitted/Protected Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.1270856872	<0.001
$V_L \times V_{Th} \times N$	0.0000038735	<0.001

Table 12 indicates that both coefficients are statistically significant at the significance level of 0.05. The form of the model (with a 10-digit decimal precision) would be:

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = -4.1270856872 + 0.0000038735(V_L \times V_{Th} \times N)$$

or alternatively expressed in terms of Number of Crashes:

$$\text{Number of Crashes} = e^{-4.1270856872 + 0.0000038735(V_L \times V_{Th} \times N) + \ln(6)}$$

### Step 2: Statistical analysis by including the main effects in the model

The results of the statistical analysis in terms of statistical significance are presented in Table 13.

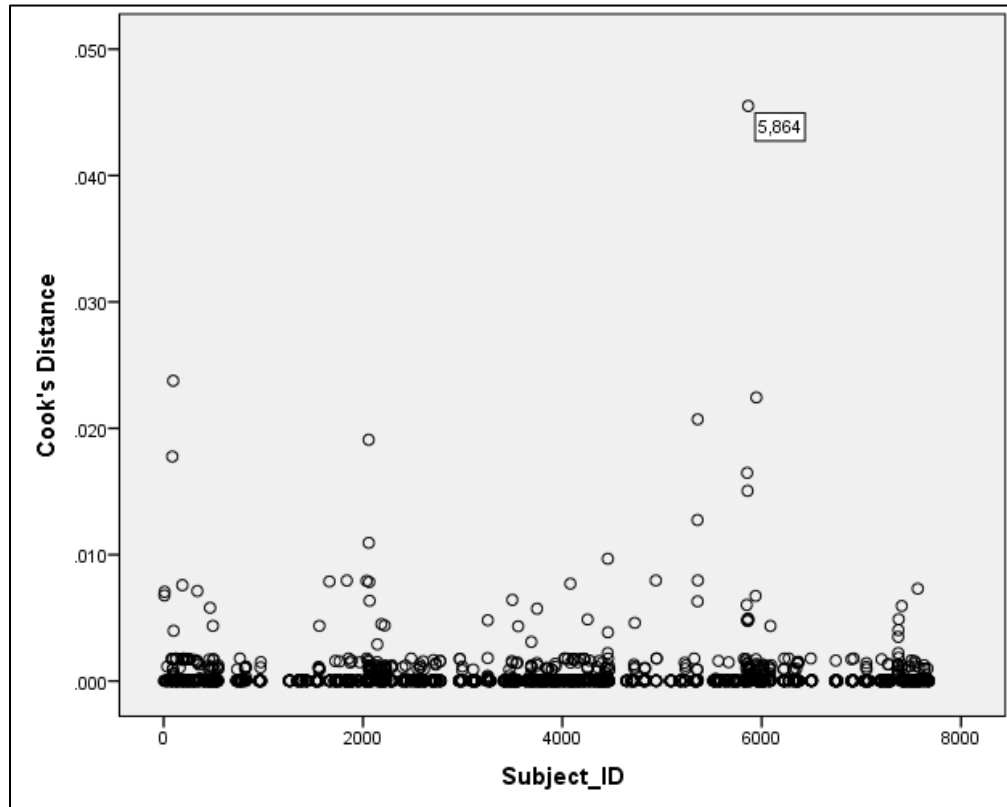
**Table 13: Parameter Estimates for Initial Model of Permitted/Protected Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.4810545812	<0.001
$V_L \times V_Th \times N$	0.0000015270	0.077
$V_L$	0.0026513094	0.008
$V_Th$	0.0006261406	0.004

Table 13 indicates that not all coefficients are statistically significant at the significance level of 0.05.

### Step 3: Influential Points Identification

The plot of Cook's distance is presented in Figure 6. The influential individual observations, meaning that they differ appreciably from most of the values, which will be further examined in order to decide whether they should be excluded from the dataset are marked on Figure 6.



**Figure 6: Cook's D for Permitted/Protected Model**

According to Figure 6, there is only one data point (i.e., 5,864) that is marked whose Cook's D that is relatively higher compared to the other observations, meaning that further analysis is required in order to decide whether they should be excluded from the database.

*Analysis of influential point with Subject ID# 5,864*

The descriptive information of the case with Subject ID# 5,864 is presented in Table 14, whereas the aerial image of the intersection is shown in Figure 7.

**Table 14: Crash Related Descriptive Information for Influential Point #5,864**

Subject ID#	5,864
Intersection ID#	14
Direction	Left Turn from West w/ Opposing from E
# of_Crashes	6
Hour	15:15
	15:11
	15:15
	15:51
	15:59
	15:23
Date	2010-05-22
	2010-06-16
	2012-03-13
	2012-04-17
	2012-10-30
	2014-10-30
L_V	46
Th_V	475
Type_LT_P	Permitted/Protected
N_Opp_Th_Lanes	2
Latitude	38.0183
Longitude	-84.4649

According to Table 14, there seems to be an issue of independence among the crash occurrences. The first two crashes, as well as the third and fourth, occurred within a one-month interval; as mentioned in the previous cases, this might indicate that it was these specific time periods that lead to these crashes rather than the transportation-wise characteristics of the approaches themselves. As in the previous case, the time interval of the crash occurrences belongs in the PM rush hour period of the day, but in this case between 3 to 4 pm instead of 5 to 6 pm. Once again, the latter fact supports the idea that the time interval might independently affect a crash occurrence. For example, this might be related to a function of driving behavior aggressiveness throughout the day.

Figure 7 reveals an additional potentially contributing element. There is a commercial development driveway from which vehicles can exit and enter. It might be likely that crashes that have been reported as opposing volume from East to actually correspond to the “exits” from the commercial development. Also, the fact that the crashes occurred from 3 to 4 pm might be associated to the “rush hour” of the commercial development, meaning that there are many customers leaving from the supermarket in a fast manner and rather inattentive.





**Figure 7: Influential Data Point with Subject ID# 5,864**

**Final Decision:** Exclude the data point from the dataset due to the combination of issues of independence, inflated number of crash occurrences, and ambiguity in terms of data collection

**Step 4: Re-run the model with excluded influential points**

The results of the statistical analysis in terms of statistical significance are presented in Table 15.

**Table 15: Parameter Estimates for Initial Model of Permitted/Protected Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.0982683003	<0.001
$V_L \times V_{Th} \times N$	0.0000033242	<0.001

Table 15 indicates that both coefficients are statistically significant at the significance level of 0.05. The form of the model (with a 10-digit decimal precision) would be:

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = -4.0982683003 + 0.0000033242 (V_L \times V_{Th} \times N)$$

or alternatively expressed in terms of Number of Crashes:

$$\text{Number of Crashes} = e^{-4.0982683003+0.0000033242(V_L \times V_{Th} \times N) + \ln(6)}$$

### Step 5: Assumption Assessment

The Goodness of Fit (GOF) statistics are provided in Table 16.

**Table 16: GOF Statistics for Permitted/Protected Left-Turn Signal Scheme**

	Value	df	Value/df
<b>Deviance</b>	1045.338	2425	0.431
<b>Pearson Chi-Square</b>	2479.306	2425	1.022
<b>Log Likelihood</b>	-970.574		
<b>Akaike's Information Criterion (AIC)</b>	1947.148		
<b>Bayesian Information Criterion (BIC)</b>	1964.534		

Table 16 indicates that the Pearson's Chi-square is 2479. The critical value for a Chi-square with 2425 degrees of freedom is 2541 at the significance level of  $\alpha = 0.05$ . Therefore, the Pearson's Chi-Square, as it was calculated from the regression model, does not belong in the rejection region since  $2479 < 2541 = \text{critical value}$ ; therefore, there is not statistically significant evidence to reject the null hypothesis. This means that we cannot conclude that the probability distribution is not a Negative Binomial. Just for the sake of completeness, the corresponding p-value is also calculated as  $p - \text{value} = 0.22 \gg 0.05$  which is much greater than 0.05. Moreover, the dispersion is 1.022 which is almost 1, meaning that practically all of the overdispersion has been accounted for in the model.

### Step 6: Compare Negative Binomial with Poisson Regression

The statistical test that tests whether the Negative Binomial regression provides a significant improvement in the model compared to the Poisson regression is conducted in SAS and the result is presented in Table 17.

**Table 17: Statistical Comparison Between Negative Binomial and Poisson Regression**

Parameter	Chi-Square	p-value
Dispersion	17.1020	<0.0001



According to Table 17,  $p - value < 0.0001 < 0.05$ ; therefore, there is statistical significant evidence at the significance level of  $\alpha = 0.05$  to reject the null hypothesis and accept the alternative. In other words, the Negative Binomial distribution is a more suitable distribution compared to the Poisson.

### Nomograph Creation

There will be one nomograph created for the Permitted-only left-turn signal scheme with two distinct curves: one curves corresponds to the case where there is 1 opposing through lane, whereas the second curves corresponds to the case where there are 2 opposing through lanes. In addition, the nomograph assumes one crash per year. Therefore, the nomograph actually corresponds to the following equation:

$$\text{Number of Crashes} = e^{-4.0982683003 + 0.0000033242(V_L \times V_{Th} \times N) + \ln(6)}$$

$$1 = e^{-4.0982683003 + 0.0000033242(V_L \times V_{Th} \times N) + \ln(6)}$$

$$-4.0982683003 + 0.0000033242(V_L \times V_{Th} \times N) + \ln(6) = 0$$

$$V_L = \frac{4.0982683003 - \ln(6)}{0.0000033242(V_{Th} \times N)}$$

The nomograph for the Permitted/Protected left-turn signal scheme is presented in Figure 8.

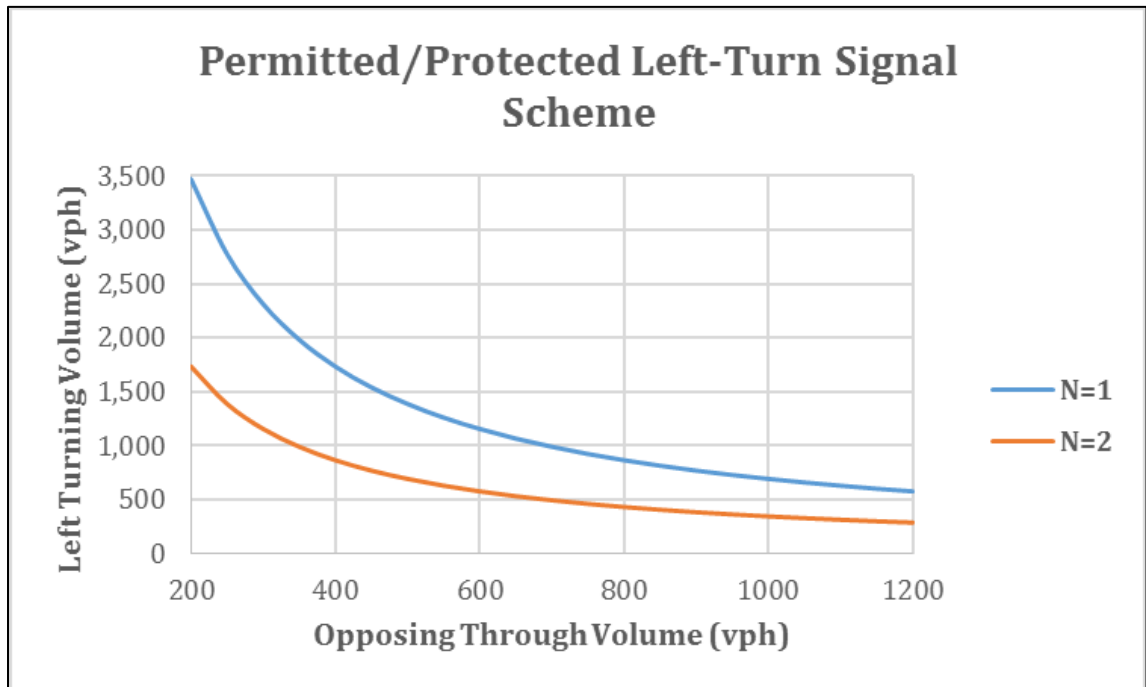


Figure 8: Nomograph for Permitted/Protected Left-Turn Signal Scheme

### 5.3 Model 3: Protected Left Turn Signal Scheme

#### Step 1: Statistical analysis by including only the 3-way interaction term

The results of the statistical analysis in terms of statistical significance are presented in Table 18.

**Table 18: Parameter Estimates for Initial Model of Permitted-only Left-Turn Signal Scheme**

Parameter	Coefficient Estimate	p-value
Intercept	-4.4889513102	<0.001
$V\_L \times V\_Th \times N$	0.0000022776	<0.001

Table 18 indicates that both coefficients are statistically significant at the significance level of 0.05. The form of the model (with a 10-digit decimal precision) would be:

$$LN\left(\frac{\text{Number of Crashes}}{6}\right) = -4.4889513102 + 0.0000022776(V\_L \times V\_Th \times N)$$

or alternatively expressed in terms of Number of Crashes:

$$\text{Number of Crashes} = e^{-4.4889513102+0.0000022776(V\_L \times V\_Th \times N)+\ln(6)}$$

#### Step 2: Statistical analysis by including the main effects in the model

The results of the statistical analysis in terms of statistical significance are presented in Table 19.

**Table 19: Parameter Estimates for Initial Model of Permitted-only Left-Turn Signal Scheme**

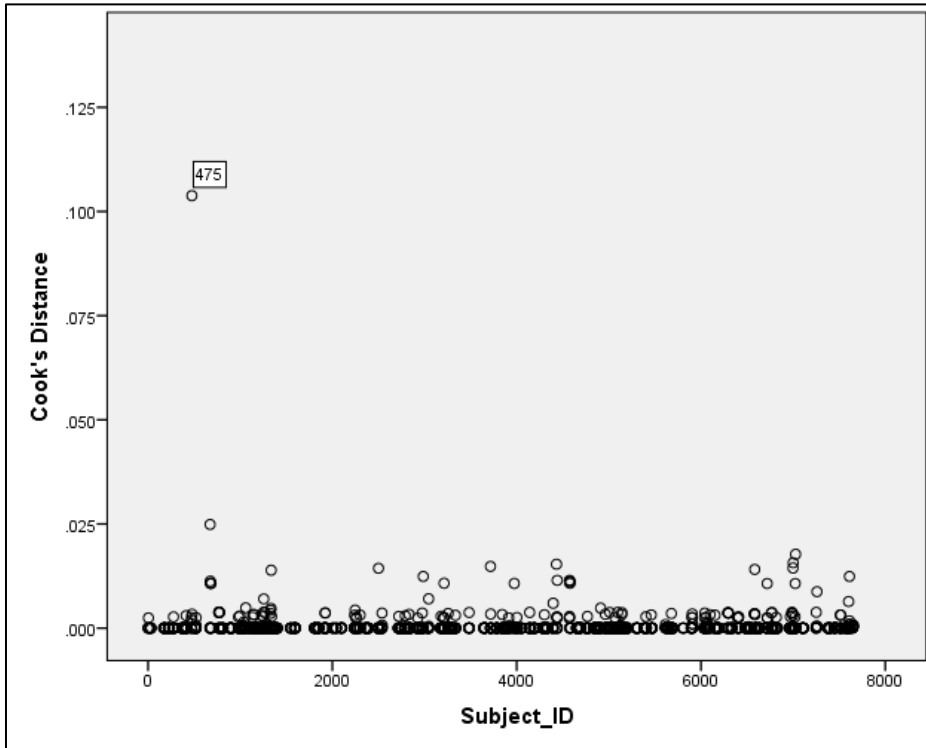
Parameter	Coefficient Estimate	p-value
Intercept	-4.4810545812	<0.001
$V\_L \times V\_Th \times N$	0.0000015270	0.455
$V\_L$	0.0026513094	0.003
$V\_Th$	0.0006261406	0.300

Table 19 indicates that not all coefficients are statistically significant at the significance level of 0.05.

#### Step 3: Influential Points Identification

The plot of Cook's distance is presented in Figure 9. The influential individual observations, meaning that they differ appreciably from most of the values, which will be

further examined in order to decide whether they should be excluded from the dataset are marked on Figure 9.



**Figure 9: Cook's D for Protected Model**

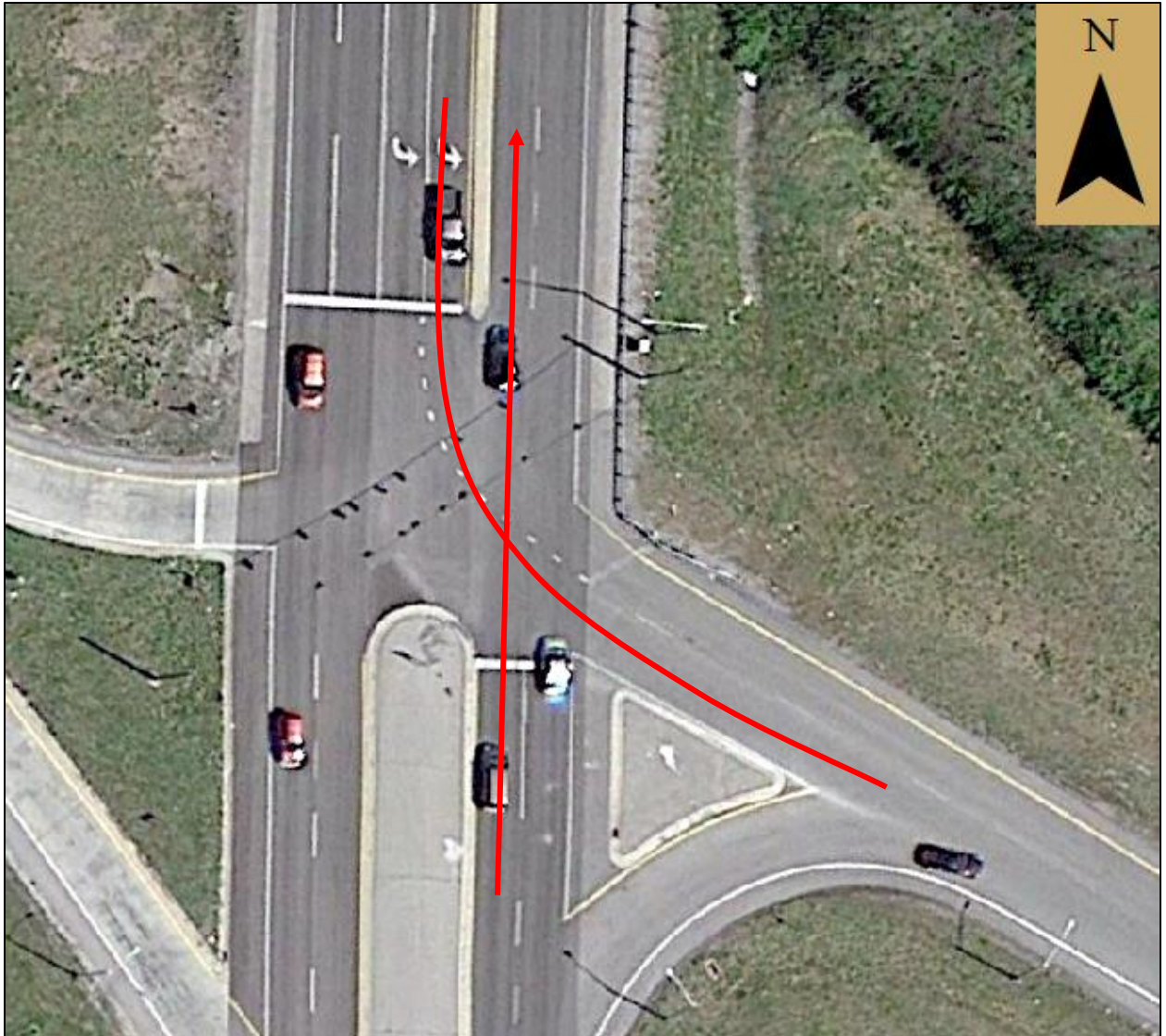
According to Figure 9, there is only one data point (i.e., 475) that is marked whose Cook's D that is relatively higher compared to the other observations, meaning that further analysis is required in order to decide whether they should be excluded from the database.

*Analysis of influential point with Subject ID# 475*

The descriptive information of the case with Subject ID# 475 is presented in Table 20, whereas the aerial image of the intersection is shown in Figure 10.

**Table 20: Crash Related Descriptive Information for Influential Point #475**

Subject ID#	475
Intersection ID#	64
Direction of Crash	Left Turn from North w/ Opposing from South
Number of Crashes	0
Left Turning Volume	557
Opposing Through Volume	1377
Type of Left Turn Signal Scheme	Protected
Number of Opposing Through Lanes	2
Latitude	38.2239
Longitude	-84.5391



**Figure 10: Influential Data Point with Subject ID# 475**

According to Table 20, the left-turning volume seems to be fairly large; however, this is not considered a reason for exclusion. As far as independence is concerned, since there is only one crash occurrence there is not an independence issue either. The intersection geometry indicates that there are two exclusive left turning lanes. In general, this is relatively rare and therefore this explains the inflated left-turn volume which in turn makes this data point influential.

**Final Decision:** Do not exclude the data point, but keep in mind that when there are two exclusive left-turning lanes, this might affect the number of crashes that might occur. This should be further analyzed in future research.

#### **Step 4: Re-run the model with excluded influential points**

Since no data point was excluded from the dataset, the results of the statistical analysis are the same as in Table 18.

### Step 5: Assumption Assessment

The Goodness of Fit (GOF) statistics are provided in Table 21.

**Table 21: GOF Statistics for Permitted-only Left-Turn Signal Scheme**

	Value	df	Value/df
<b>Deviance</b>	972.835	1889	0.515
<b>Pearson Chi-Square</b>	1950.587	1889	1.032
<b>Log Likelihood</b>	-1070.849		
<b>Akaike's Information Criterion (AIC)</b>	2152.894		
<b>Bayesian Information Criterion (BIC)</b>	2171.427		

Table 21 indicates that the Pearson's Chi-square is 1951. The critical value for a Chi-square with 1889 degrees of freedom is 1991 at the significance level of  $\alpha = 0.05$ . Therefore, the Pearson's Chi-Square, as it was calculated from the regression model, does not belong in the rejection region since  $1951 < 1991 = \text{critical value}$ ; therefore, there is not statistically significant evidence to reject the null hypothesis. This means that we cannot conclude that the probability distribution is not a Negative Binomial. Just for the sake of completeness, the corresponding p-value is also calculated as  $p\text{-value} = 0.15 > 0.05$  which is greater than 0.05. Moreover, the dispersion is 1.032 which is almost 1, meaning that practically all of the overdispersion has been accounted for in the model.

### Step 6: Compare Negative Binomial with Poisson Regression

The statistical test that tests whether the Negative Binomial regression provides a significant improvement in the model compared to the Poisson regression is conducted in SAS and the result is presented in Table 22.

**Table 22: Statistical Comparison Between Negative Binomial and Poisson Regression**

Parameter	Chi-Square	p-value
Dispersion	15.79999	<0.0001

According to Table 22,  $p\text{-value} < 0.0001 < 0.05$ ; therefore, there is statistical significant evidence at the significance level of  $\alpha = 0.05$  to reject the null hypothesis and accept the alternative. In other words, the Negative Binomial distribution is a more suitable distribution compared to the Poisson.

### *Nomograph Creation*

There will be one nomograph created for the Permitted-only left-turn signal scheme with two distinct curves: one curve corresponds to the case where there is 1 opposing through lane, whereas the second curve corresponds to the case where there are 2 opposing through

lanes. In addition, the nomograph assumes one crash per year. Therefore, the nomograph actually corresponds to the following equation:

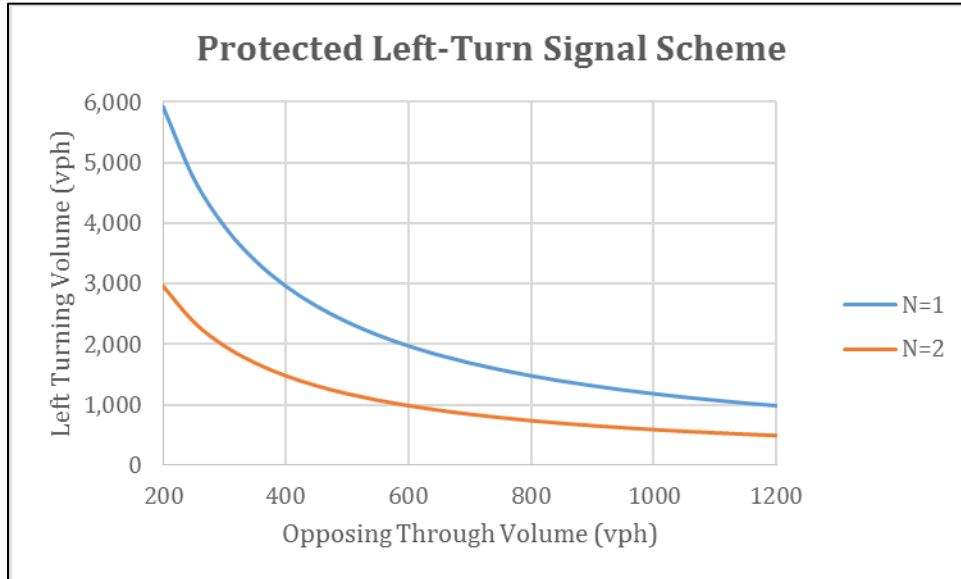
$$\text{Number of Crashes} = e^{-4.4889513102 + 0.0000022776(V_L \times V_{Th} \times N) + \ln(6)}$$

$$1 = e^{-4.4889513102 + 0.0000022776(V_L \times V_{Th} \times N) + \ln(6)}$$

$$-4.4889513102 + 0.0000022776(V_L \times V_{Th} \times N) + \ln(6) = 0$$

$$V_L = \frac{4.4889513102 - \ln(6)}{0.0000022776(V_{Th} \times N)}$$

The nomograph for the Protected left-turn signal scheme is presented in Figure 11.



**Figure 11: Nomograph for Permitted/Protected Left-Turn Signal Scheme**

## 6 GUIDELINES

The separate models developed for the permitted and permitted/protected phase types in order to determine a model that could best describe each type can be combined to develop decision models regarding the left-turn phasing selection. Equation 1 corresponds to the crash prediction model for the permitted-only left-turn phasing, whereas Equation 2 corresponds to the crash prediction model for the permitted/protected left-turn phasing. It is worth mentioning that models corresponding to the protected-only phase have also been developed, but mainly for the sake of completeness and statistical evaluation. The protected-only regression line has no practical use in the warrant or nomograph development for the decision-making of the left-turn phasing type. The permitted/protected phase is actually the criterion for the establishment of the protected-only phase; if the left and through volume combination is above the permitted/protected regression line, then the specific intersection approach should automatically be operated by a protected-only left-turn phase.

At this point it is repeated that the crash data correspond to a 6-year period and this should be accounted for in the Negative Binomial regression model. This was achieved by utilizing the offset variable, which actually normalizes the number of crashes, through a time rate, to a one-year period. The one-year period approach is desirable in order for the results to be more comparable to other existing or future studies and more flexible in terms of suggesting warrants or policies in general.

$$\begin{aligned} \text{Number of Crashes}_{\text{Permitted-only}} \\ = e^{-4.4769746934+0.0000079622 (V_L \times V_{Th} \times N) + \ln(6)} \end{aligned} \quad (1)$$

$$\text{Number of Crashes}_{\text{Perm/Prot}} = e^{-4.0982683003+0.0000033242 (V_L \times V_{Th} \times N) + \ln(6)} \quad (2)$$

where:

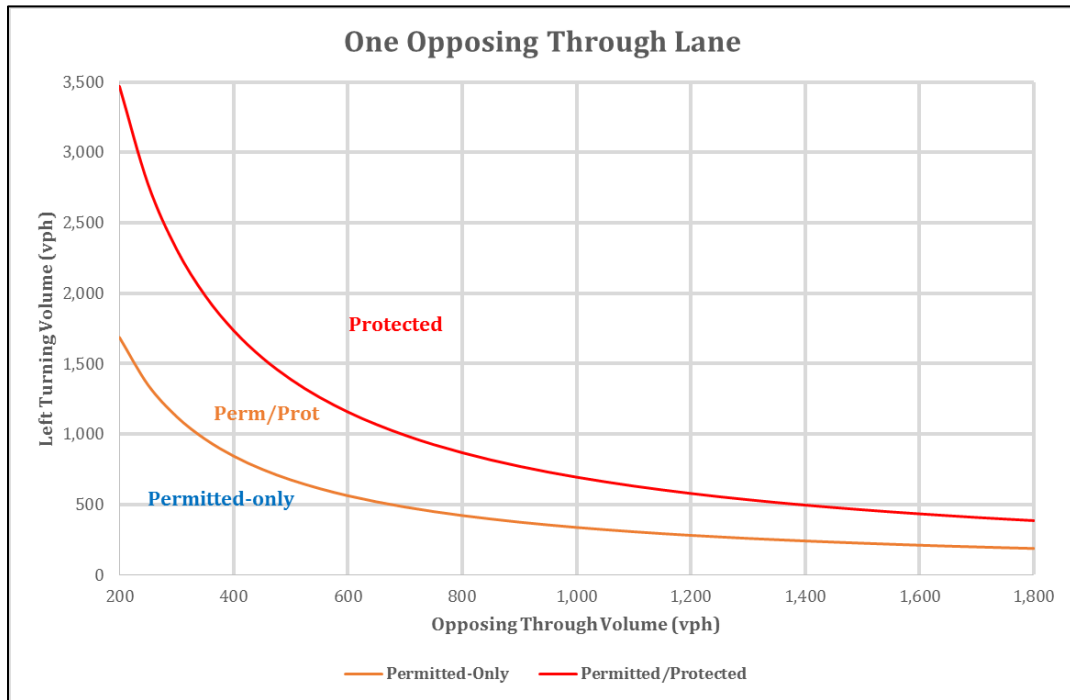
$V_L$  : Volume of Left-Turns

$V_{Th}$  : Respective Opposing Through Volume

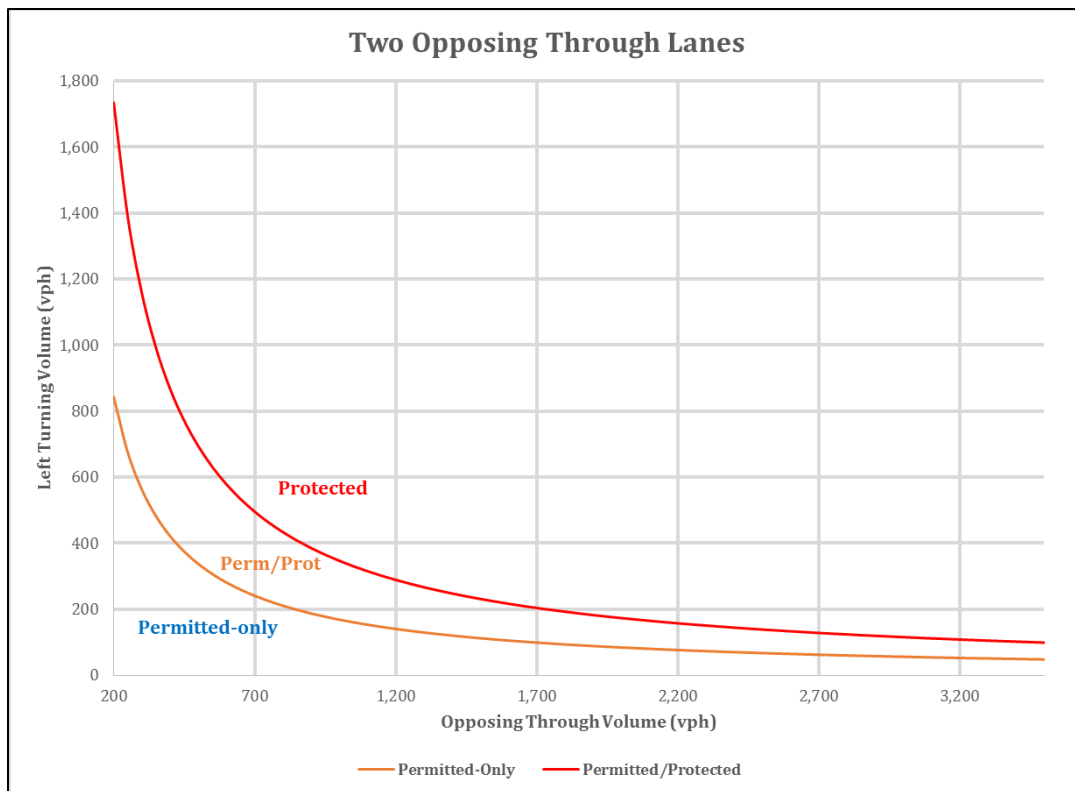
$N$  : Number of Opposing Through Lanes

Based on the models shown in Equations 1 and 2, a series of guidelines were developed to assist in left turn phase selection representing the thresholds between phase selections. Figures 12 and 13 are examples of such graphs. These figures show a line of equality where the combination of left turn volume, opposing volume and signal phasing equate to a single crash per year for the evaluated hour based on the explanatory model. Guidelines similar to these may be used in selection of appropriate left turn phasing by identifying a crash threshold (such as 1 crash shown in the graph below). Combinations below the solid line would result in a recommended permitted phase, combinations above the dashed line result in a protected-only phase, while combinations between the two lines will result in permitted/protected phase.

The final guidelines that are suggested based on this study are provided in Figures 12 and 13.



**Figure 12: Guidelines for One Opposing Through Lane**



**Figure 13: Guidelines for Two Opposing Through Lanes**



Finally, the variance explained in the two models is reported; in an ordinary linear regression the variance explained in a regression model is well defined by the  $R^2$ . However, the  $R^2$  statistic does not extend generally to the Poisson or Negative Binomial regression or count data. A metric that can potentially be utilized in order to determine the variance explained in the model for Generalized Linear Models (GLMs) is the pseudo  $R^2$  (31). It is noted that there are several R-squared measures such as the Efron's, McFadden's, Cox & Snell's, Nagelkerke/Cragg & Uhler's, and McKelvey & Zavoina's among others; however, not all of them are direct equivalents to the percentage of variance explained in the model. In this case, Efron's pseudo- $R^2$  is calculated and reported because it is a typical selection and most importantly because it can be interpreted as the variance explained by the model.

The pseudo- $R^2$  for the Permitted-only model (Equation 1) is 0.648, whereas the pseudo- $R^2$  for the Permitted/Protected model (Equation 2) is 0.561. Both values regarding the pseudo- $R^2$  values for the two models are considered acceptable.

## 7 DISCUSSION AND CONCLUSIONS

The primary measure to control left turns, perhaps the most critical intersection movement in terms of safety, is the implementation of a variety of signal phasing schemes including permitted-only, protected-only, or a combination of permitted/protected left turns. Many state DOTs have developed warrants or guidelines for selecting the left turn phasing type for an intersection based on a number of explanatory factors such as the cross product between left turn movements and opposing through volumes. The purpose of this research was to create a left-turn phasing guidance at signalized intersections which determines the most appropriate left-turn phasing scheme in each case based on a combination of left and opposing volumes and the number of opposing through lanes. The number of crashes, during the 6-year period 2010-2015, was considered the dependent variable in the negative binomial regression model.

An important area of discussion is variable selection and the combination of such variables, particularly the cross product between opposing through movements and left turn maneuvers. For example, an intersection with 500 left turning vehicles and 1,000 opposing through vehicles has the same cross product as an intersection with 1,000 left turning vehicles and 500 opposing through vehicles. It is therefore reasonable to assume that each combination would have a different safety performance and this was captured in the model developed here though the inclusion of the separate values and their relative impact, i.e., left turn volume has a power of 1.5 in the model. Based on the results, it is determined that left turning movements have a higher correlated relationship with crashes than the opposing through movements. The use of the cross product as a criterion for the selection of the left-turn phase has been previously questioned (1, 32). This was based not only on the relative importance of the contributing volumes but on the use of a single threshold value that mainly reflects peak hour (or period) conditions.

The purpose of the research was to develop a predictive tool for left turn crashes and to be used as guidance for determining appropriate left-turn phasing based on safety. Figures 12 and 13 are references that could be used to assist in signal phasing decisions. Simply knowing the typical left turn demand, opposing through volume, and opposing number of lanes, designers are able to determine the left-turn phasing that could result in one crash per year. Based on engineering judgment, conflict thresholds would then be determined as a distinction between permissive and some form of protected phasing. These figures illustrate the differences between one and three opposing lanes indicating that a higher number of left turns can be accommodated at permitted phasing with one opposing lane than in three opposing lanes. Graphs similar to these are easily derived for any number of anticipated crashes based on agency preferences. Designers will have to balance the operational and safety aspects of their choice in order to determine the appropriate left-turn phasing to be used.

The proposed nomographs could be considered as an improvement of the existing guidelines transportation agencies use currently. Most of the guidance is based on a single element, e.g. left turn volume, cross product, number of crashes over a period of time, and number of opposing lanes. The proposed approach combines all these criteria into a single

concept and allows designers consider the interactions of the criteria simultaneously. Another advantage of these nomographs is the ability to develop left-turn phase decisions based on time of day traffic volumes. Current practices utilizing the existing criteria for phase selection consider peak traffic operations and as such they impose a phasing plan based on these volumes. The proposed nomographs can address the hourly traffic variations and provide for a more dynamic phasing plan to deal with these volume changes.

The findings of this study indicate that additional work is needed to improve understanding of the left-turn phasing implications. As a first step, the combination of the criteria developed here and operational efficiency nomographs need to be combined to achieve a balanced solution that could efficiently address both safety and operations. The latter can be actually realized as a multi-objective optimization problem. The development of guidance for time of day operations need to be further examined, in order to provide for a more robust decision tool. It would be desirable to consider the time of day as part of the model variables but the current database does not allow for this. Another issue is the potential to compare these models with other surrogate safety measures, such as those produced through micro-simulation and determine the potential for utilizing these surrogate measures to understand the intersection-wide potential issues associated with left-turn phasing decisions. In addition, confound variables should be measured and included in the model; for example, the distance that a vehicle must cover in order to complete a left turn might be considered explanatory variable in the model. This distance can be measured in Google Earth with a relatively acceptable precision for the needs of the study.

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## VITA

### NAME

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Kiriakos Amiridis

### PLACE OF BIRTH

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Queens, New York, USA.

### EDUCATION

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**University of Kentucky:** (August 2015 – December 2016)

**M.Sc. in Civil Engineering**

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**GPA:** 4.00/4.00 (Excellent)

**Master Thesis:** «Safety-Based Guidelines for Left-Turn Phasing Decisions with Negative Binomial Regression »

**Supervisor:** Dr. Nikiforos Stamatiadis

**National Technical University Of Athens:** Athens, Greece (September 2007- June 2014)

**B.Sc. in Surveying Engineering** – 5 Year Curriculum

**Major in Transportation Engineering**

**GPA:** 8.0/10.0 (Very Good)

**Diploma Thesis:** «3-D Road Design By Applying Differential Geometry and B-Spline Interpolation Curves»; Grade: 10.00/10.00 (Excellent)

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**Apolytirion** - 4th Unified Lyceum of Alimos, Alimos, Greece (June 2007)

GPA: 19.0/20.0 (Excellent)

### PROFESSIONAL EXPERIENCE

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- Research Assistant under Professor Nikiforos Stamatiadis at the University of Kentucky (August 2015 – present)
- Research Assistant under Professor Nikiforos Stamatiadis; Project MRI2: Integrated Simulation and Safety for South-eastern Transportation Center, Knoxville, Tennessee (January 2016 – August 2016).
- Teaching Assistant at the University of Kentucky at CE 531 Geometric Design of Roadways (Fall 2016)
- Teaching Assistant at the University of Kentucky at CE 331 Transportation Engineering (Fall 2015)

- National Technical University of Athens (June 2011 – July 2011); Land Surveying Works for the Inclusion of a Section of the Nisyros Island in the National Cadastre
- Summer Internship at the National Bank of Greece (May 2008 – October 2008)

## PUBLICATIONS

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- **Amiridis, K.**, Stamatiadis, N., Kirk, A., *Safety-Based Signalized Intersection Left-Turn Phasing Decisions*, Journal of the Transportation Research Board (TRR), 2017.
- **Amiridis, K.**, Psarianos, B., *Calculation of the available 3-d sight distance by modeling the roadway as a 3-d B-spline surface*, Advances in Transportation Studies an International Journal, 2016 Special Issue, Vol. 2.
- **Amiridis, K.**, Psarianos, B., *Application of Differential Geometry to Solve the Highway Alignment Location Problem*, Scientific Journal of the Orenburg State University (Vestnik OGU), 10(171), 262-270, 2014 Oct.
- **Amiridis, K.**, Psarianos, B., *Three Dimensional Road Design By Applying Differential Geometry and Conventional Design Approach Criteria*, Mathematical Design & Technical Aesthetics, 2015, Vol. 3, No1, pp 46-75.

## REFERRED CONFERENCE PROCEEDINGS

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- Stamatiadis, N., Sturgill, R., **Amiridis, K.**, *Benefits from Constructability Reviews*, World Conference on Transportation Research (WCTR), July 2016.
- **Amiridis, K.**, Psarianos, B., Stamatiadis, N., *Generic Methodology for 3-D Available Sight Distance Calculation*, ASCE International Conference on Transportation & Development (ICTD), June 2016.
- **Amiridis, K.**, Psarianos, B., *A Direct and Accurate Sight Distance Calculation by Simulating the Road as a Three-Dimensional B-Spline Surface*, 5<sup>th</sup> International Road Safety and Simulation Conference (RSS), 2015.
- **Amiridis, K.**, Psarianos, B., *Direct Calculation of Water Film Paths as Geodesic Curves on a three-Dimensional Road Surface to address Hydroplaning*

*Phenomena*, 5<sup>th</sup> International Symposium of Highway Geometric Design (ISHGD), June 2015.

- **Amiridis, K.**, Psarianos, B., *Three Dimensional Road Design By Applying Differential Geometry and Conventional Design Approach Criteria*, 94<sup>th</sup> Annual Transportation Research Board Meeting (TRB), 2015.

## **AWARDS & DISTINCTIONS**

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- Awarded a fellowship to obtain a Master's in Civil Engineering Program at the University of Kentucky.
- Best Thesis Award from the Department of Rural and Surveying Engineering at the National Technical University of Athens for the year 2014.
- Award of Excellence (over 92.5%) for exceptional student performance and Award of Highest Grade in Class throughout all the classes of Gymnasium (Junior High) and Lyceum (High School) (2002, 2003, 2004, 2005, 2006, 2007)
- Selected from all over Greece to monitor the 7<sup>th</sup> Summer School of Astronomy (August 2006)
- Distinction in the Qualifying Competition «Euclid» of the Greek Mathematical Society (2005).

## **PROFESSIONAL AFFILIATIONS**

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- Younger Member in the Street and Highway Operations Committee of the Transportation & Development Institute (T&DI)
- Member of the American Society of Civil Engineers (ASCE)
- Member of Chi Epsilon Civil Engineering Honor Society, Kentucky Chapter
- Member of the Institute of Transportation Engineers (ITE), UK Student Chapter and International
- Member of Engineers Without Borders Design Team, Kentucky Chapter
- Friend of the AFB10 Geometric Design Committee of the Transportation Research Board (TRB)